Time and symbolism: two hidden faces of the prime numbers

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1 Introduction

We analyze the consequences derived from considering natural numbers in association with time states, which in a first intuitive approach would be equivalent to examining different counting methods depending on the wider or narrower time interval between the instant in which we count a natural number and its successor. We will use the lecture [2] and its complete theoretical development [3] as the main references, in which, using dynamic processes, we construct a family of first-order arithmetic interpretations so that an acceleration characterizes an arithmetic statement $\mathscr{S}$, a characterization which is lost in a time instant, thus obtaining a temporal singularity. As might be expected, the subject will be focused on the prime number concept. Actually, the $\mathscr{S}$ statement which we refer to is the Goldbach Conjecture (any given even number greater than two is the sum of two prime numbers), a statement that, to this date, has never been proven or disproven. Furthermore, in his famous speech at the meeting of the mathematical society of Copenhagen in 1921, G.H. Hardy stated that $\mathscr{S}$ is probably as difficult as any of the unsolved problems in mathematics and therefore the Goldbach problem is not only one of the most famous and difficult problems in number theory, but also in the whole of mathematics ([6]). However, it is not our goal to analyze $\mathscr{S}$ deeply but rather to see what a dynamic (and notational) aspect of arithmetic can contribute to the explanation of the unfathomable mystery of prime numbers.

2 Dynamic aspect

In [5], J.J. Sylvester declares:

> *I have sometimes thought that the profound mystery which envelops our conceptions relative to prime numbers depends upon the limitations of our faculties in regard to time, which like space may be in*

[1]Until the 2008-2009 academic year. I am now devoted to investigation.
[2]Idem.

> *essence poly-dimensional and that this and other such sort of truths would become self-evident to a being whose mode of perception is according to* superficially *as opposed to our own limitation to* linearly *extended time.*

Regardless of what was going through the mind of this mathematical genius when he wrote that passage, we state what has been proven in [3].

There is a family of movements given by a real function $s = s(v, t)$ where $0 < v \leq 1$ is a parameter, $t \geq 0$ represents time and s space. For $v = a$ we choose $0 = t_{a0} < t_{a1} < t_{a2} < \ldots$, which represent instants of time corresponding to the natural numbers $0, 1, 2, \ldots$, respectively. Similarly, for $v = b$ we choose the instants $0 = t_{b0} < t_{b1} < t_{b2} < \ldots$ corresponding to $0, 1, 2, \ldots$ respectively, a process which may be generalized to $0 = t_{v0} < t_{v1} < t_{v2} < \ldots$ for all $0 < v \leq 1$. We should specify what we mean by choosing those time instants corresponding to natural numbers. The $s = s(v, t)$ function has been constructed for all $0 < v \leq 1$ using a previously constructed bijection $\psi_v : \mathbb{R}^+ \to [0, A_v)$ with $A_v \in (0, +\infty]$, which allows the choice of $t_{vn} = \psi_v(n)$ in the movement corresponding to v for any natural number n.

Each bijection ψ_v allows us to naturally embed the usual arithmetic of $\mathbb{N} = \{0, 1, 2, \ldots\}$ into $[0, A_v)$. For example, if v verifies that $\psi_v(2) = 0.83$, $\psi_v(3) = 1.725$, $\psi_v(5) = 4.3$, $\psi_v(6) = 8.95$, since $2 + 3 = 5$ and $2 \times 3 = 6$, when we embed the arithmetic we obtain the operations $0.83 \oplus_v 1.725 = 4.3$ and $0.83 \otimes_v 1.725 = 8.95$. Thus, through ψ_v, we obtain an arithmetic that is isomorphic to the usual one by means of a simple notational change.

Let us return to the $\mathscr{S}$ statement and consider an even number $\alpha \geq 16$. This condition is established for technical demonstration reasons (for $2 < \alpha < 16$ the veracity of $\mathscr{S}$ is trivially verifiable). We also consider the condition $\alpha - 3$ not prime (if it were prime, $\alpha = 3 + (\alpha - 3)$ and $\mathscr{S}$ would be true) and the condition $\alpha/2$ not prime (if it were, $\alpha = \alpha/2 + \alpha/2$ and $\mathscr{S}$ would also be true). The $\mathscr{P}$ set of even numbers $\alpha \geq 16$ that satisfy the conditions $\alpha - 3$ and $\alpha/2$ not prime is infinite and we focus our attention on it. Clearly, for $\alpha \in \mathscr{P}$ the statement $\mathscr{S}$ will be true if and only if there is a prime k with $5 < k < \alpha/2$ such that $\alpha - k$ is prime. For each fixed $0 < v \leq 1$ the law of movement $s_v(t) = s(v, t)$ has a continuous acceleration, and a pair of real coefficients $x_{k,v}, y_{k,v}$ will appear in the formula that determines it for each natural $4 \leq k < \alpha/2$, that is, a sequence of points $P_{k,v} = (x_{k,v}, y_{k,v})$, which we call *essential points*. We verify that for each $0 < v < 1$, k and $\alpha - k$ are prime if and only if $P_{k,v} = P_{k-1,v}$, or equivalently if two consecutive essential points are repeated. However, for $v = 1$ all essential points coincide and the previous characterization is lost.

Let us now consider v, a time variable. We have a movement of movements. That is, for each v instant a $s_v(t)$ movement is defined and these vary continuously with respect to v. From the consideration $s = s(v, t)$ as a movement in *bi-dimensional* time we extract the following consequences:

(i) The simultaneous primality of k and $\alpha - k$ is equivalent to an equality of certain elements of $\mathbb{R}^2$. (ii) There is a characterization of at least one arithmetic statement which depends on time. (iii) This arithmetic's dynamic aspect adds information to the arithmetic of Peano, which is purely static.

Note that we have used the time instant identification with real numbers in the mathematical continuum constructed via Cauchy sequences or Dedekind cuts. This identification would not be possible for Brouwer [1], for whom the only *a priori* continuous element is time.

> *How then do assertions arise which concern, not all natural, but all real numbers, i.e., all values of a real variable? Brouwer shows that frequently statements of this form in traditional analysis, when correctly interpreted, simply concern the totality of natural numbers. In cases where they do not, the notion of sequence changes its meaning: it no longer signifies a sequence determined by some law or other, but rather one that is created step by step by free acts of choice, and thus remains in statu nascendi. This becoming selective sequence represents the continuum, or the variable, while the sequence determined ad infinitum by a law represents the individual real number falling into the continuum. The continuum no longer appears, to use Leibniz language, as an aggregate of fixed elements but as a medium of free becoming.* (H. Weyl [7])

3 Notational aspect

In this section we will not consider the natural numbers associated with time states, but rather what consequences in terms of arithmetical information may be drawn (specifically, primality) from the change of initial symbolism. We start from the one given in the decimal system $\mathbb{N} = \{0, 1, 2 \ldots, 10, 11, \ldots\}$ and we consider the aforementioned bijection $\psi_v : \mathbb{R}^+ \to [0, A_v)$. Denoting $\hat{n}_v := \phi_v(n)$ we obtain $\widehat{\mathbb{N}}_v := \psi_v(\mathbb{N}) = \{\hat{0}_v, \hat{1}_v, \hat{2}_v, \ldots\}$ and the usual algebraic structure $(\mathbb{N}, +, \times)$ immediately transports to $\widehat{\mathbb{N}}_v = \psi_v(\mathbb{N})$ using the operations

$\hat{m}_\nu \oplus_\nu \hat{n}_\nu = \psi_\nu(m+n)$ and $\hat{m}_\nu \otimes_\nu \hat{n}_\nu = \psi_\nu(m \times n)$. We obtain the algebraic structure $(\widehat{\mathbb{N}}_\nu, \oplus_\nu, \otimes_\nu)$ isomorphic to the initial one. Thus we may call a natural number n or $\hat{n}_\nu$. It also happens that by construction, for $\nu = 1$ we obtain $\psi_1 = id_{\mathbb{R}^+}$ (identity map on $\mathbb{R}^+$), that is to say $n = \hat{n}_1$ for every natural number and we restore the initial notation. In [3] we prove that for every $0 < \nu \leq 1$ and for every natural number $k \geq 4$ there is a real number $x_{k,\nu}$ such that:

(a) For every $0 < \nu < 1$ it is verified that $k \geq 5$ is prime if and only if $x_{k-1,\nu} = x_{k,\nu}$. (b) For $\nu = 1$ and for every $k \geq 5$ it is verified that $x_{k-1,1} = x_{k,1}$.

Observe that in the dynamic aspect we have a characterization of the prime numbers that affects each partial structure determined by $\alpha \in \mathscr{P}$ (albeit for infinite α). That is, the veracity of the statement $\mathscr{S}$ for every $\alpha \in \mathscr{P}$ depends only on the distribution of prime numbers greater than 3 and smaller than $\alpha - 3$. That is precisely the partial structure that has enabled us to build functions $s_\nu(t)$ with continuous second derivative and interpret it as a movement. Now the situation is different. In [3] we also prove that it is not possible to construct the $s_\nu(t)$ functions with such continuity, so as to satisfy the (a) and (b) conditions above. That is, we now have no movement but we do have changes in the initial notation such that for any $0 < \nu < 1$ the prime number concept is reduced to the equality of certain elements of $\mathbb{R}$, but not only for the partial structures but for all primes greater than 3. For $\nu = 1$ we restore the initial notation and primes are no longer recognized with this equality.

The $x_{k,\nu}$ numbers are obtained by means of the second derivative of the area of regions limited by conveniently chosen deformed hyperbolas. In a first approximation this would mean that it is the geometry which adds information to the models corresponding to $0 < \nu < 1$, but this is not so. Whilst it is true that the concept of the area of a region in the Euclidean plane has been used, this has only been done so as to aid intuition, and we may do without geometric considerations. In fact, it is sufficient to use the Lebesque measure μ in $\mathbb{R}^2$ ([4]). In this way, the construction of the $x_{k,\nu}$ numbers is a consequence of the $\mu(B)$ measure of certain subsets $B \subset \mathbb{R}^2$. We may thus conclude that there is at least one adequate symbolism to represent the usual structure $(\mathbb{N}, +, \times)$ (for example, the decimal system) capable of generating by itself another adequate one with more arithmetic information than the original one, specifically about primality.

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